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MATEMÁTICA CONECTADA

Pool Testing

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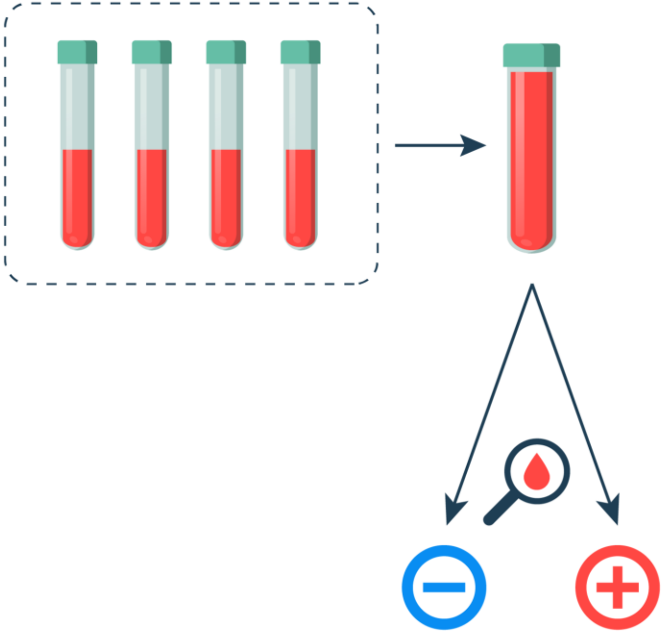
Pool Testing

Let’s review the infographic “Pool Testing”

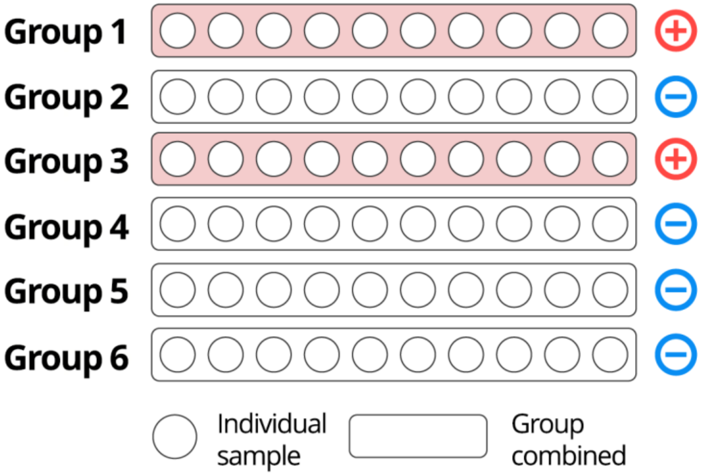
Pool Testing

- What is pool testing?
- Why is it necessary to divide each individual sample into two?
- What advantage does pool testing have over individual sample analysis?



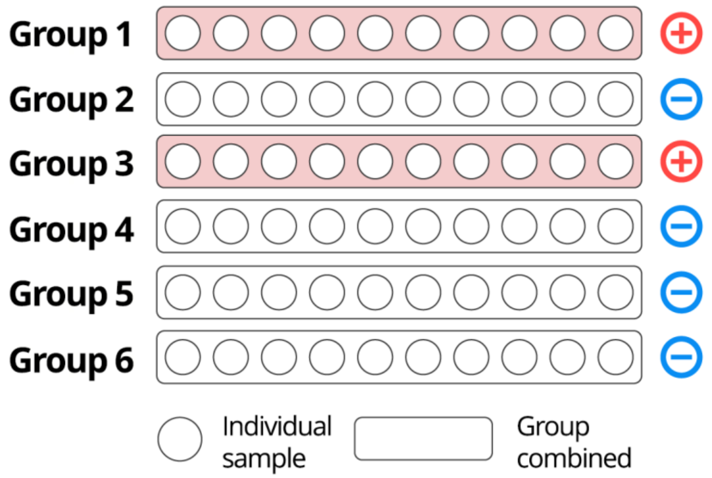
Understanding the technique

The figure depicts 60 blood samples that were pooled into six groups of 10 and then analyzed for the presence of a certain antibody. The groups that tested positive are marked in red:



Understanding the technique

- How many individual tests must be performed to detect samples with the presence of the antibody?
- How many tests in total should be performed in this situation to identify those that are positive?



Problem

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To detect steroid use in 200 athletes participating in a sports competition, their urine samples will be analyzed using the pool testing strategy. The samples will be combined in groups of 10 athletes.

It is known that the prevalence of steroid use in athletes in similar competitions, such as the 2018 Olympic Games, is 0,6%.

What is the probability that in a combined sample, at least one athlete will test positive for steroid use?

Activity 1

1. Consider the variable X = "number of positive individual samples in a combined sample" and answer:

a) What values can the variable X take?

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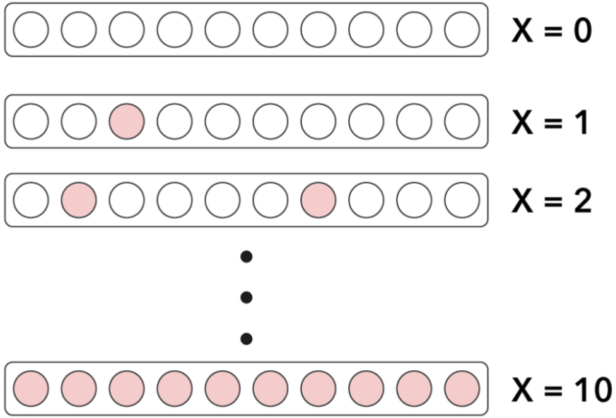

$X = \{ 0, 1, 2, 3, ...10\}$

Activity 1

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b) Why X is a random variable?

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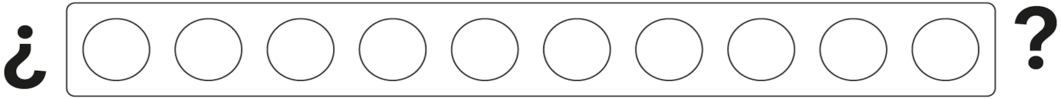
Since it is impossible to know a priori the number of positive individual samples each group will have, ***X*** is a **random variable.**



Activity 1

1. Consider the variable X = "number of positive individual samples in a combined sample" and answer:

c) Express in mathematical terms the probability corresponding to the problem question.

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What is the **probability** that in a pooled sample, **at least one** athlete tests positive for steroid use?

$$P(X \geq 1)$$

Assumption 1: Each experiment is modeled as a Bernoulli

- What do we mean by a repeated experiment in this situation?
- How many outcomes are possible in each experiment, and what are these?
- What would you represent as success and failure in this experiment?
- What are the probabilities of success and failure in this experiment?

Assumption 2: The experiments must be independent of each other

- What does it mean for experiments to be independent of each other?
- Is it reasonable to assume that an athlete's use or non-use of steroids is not affected by the use of other athletes?

Assumption 3: The probability of success is the same for each experiment.

- Is it reasonable to assume that the probability of a sample testing positive for steroids is the same for each athlete?

Activity 2

1. What is the probability that a combined sample will test positive?

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$$
\begin{aligned}
P(X \geq 1) &= 1 - P(X = 0) \\
&= 1 - \binom{10}{0} \cdot 0{,}006^0 \cdot 0{,}994^{10} \\
&\approx 1 - 0{,}94 \\
&\approx 6\ \%
\end{aligned}
$$

Activity 2

2. What is the probability that there will be more than one person who used steroids in a combined sample who tested positive?

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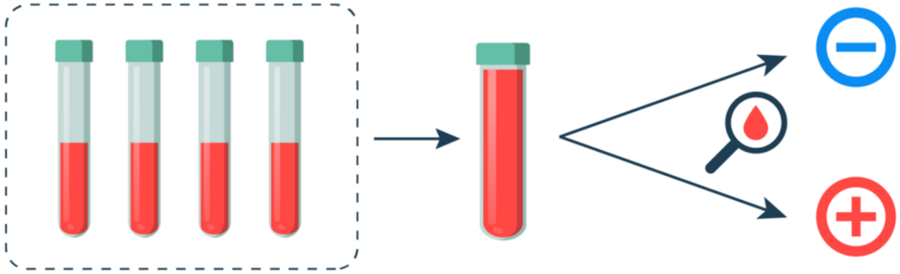
$$P(X > 1) = 1 - P(X = 0) - P(X = 1)$$

$$= 1 - 0{,}994^{10} - \binom{10}{1} 0{,}006^{1} \cdot 0{,}994^{9}$$

$$\approx 0{,}16\ \%$$

Conclusions

- Pool testing is a strategy that allows samples to be analyzed in groups, thus reducing the number of tests. It is beneficial when a large volume of samples needs to be analyzed quickly.



Conclusions

- To model a situation using the binomial distribution, it must be verified that certain assumptions are met. These are:
    - that each experiment only has **two possible complementary results**,
    - that the experiments **are independent** of each other, and
    - that the **probability of success is the same** in each experiment.

Conclusions

- Analyzing the assumptions behind using a mathematical model allows us to understand the situation better and interpret the solution to the problem.

Conclusions

- The random variable *X*, defined as "the number of successes obtained in *n* experiments", where each experiment is independent of the others and has a probability of success equal to *p*, is distributed as a **binomial of parameters *n* and *p***.

- This is usually denoted as:

$$X \sim \text{Binomial}\ (n, p)$$

Conclusions

- When X corresponds to a binomial of parameters n and p, it can take the integer values between 0 and n. The probability of obtaining $X = k$ successes is calculated by the formula:

$$P(X = k) = \binom{n}{k} \cdot p^k \cdot (1 - p)^{n-k}$$

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