MATCON

MATEMÁTICA CONECTADA

Functions Unit

Function Concepts

FACULTAD DE CIENCIAS FÍSICAS Y MATEMÁTICAS UNIVERSIDAD DE CHILE
CMMEdu Laboratorio de Educación Centro de Modelamiento Matemático
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WARM UP

In this lesson, **we will explore a type of relationship between variables called a function**.

But before need to remain some key concepts about variables

Think about your phone's battery level, as a percentage, and the amount of time it has been charging.

1. What are the variables involved in this situation?
2. What values can these variables take?
3. What happens to one variable when the other increases?

WARM UP

What we should remember for this lesson

- A variable is a feature **that can change** (e.g. battery percentage and the charging time.)

- In general, **variables can take on numerical values or qualities**. In this case, we consider both variables to be numerical: battery percentage ranges from 0% to 100%, while charging time can be measured in minutes.

- We can describe the **relationship between variables by explaining how one changes when the other changes.** For example, as time passes, the phone's battery percentage increases.

ACTIVITY 1

1

Let's go back to the earlier context, where we looked at the relationship between your phone's battery percentage and the time since it started charging.

a) In this situation, we identified two variables. Which variable depends on the other?
b) Suppose your phone was completely dead and you charge it for 20 minutes. Is it possible that at that moment the battery percentage is both 50% and 55%?
c) In general, if you charge your phone for a set amount of time, is it possible for it to show more than one battery percentage at that same moment?

Work in pairs

ACTIVITY 1

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Whole class discussion

- What feels more natural in this context: for the percentage to depend on time, or for time to depend on the percentage?
- Is it possible for the phone to show two different battery percentages at the same time? Why?

CONCLUSIONS

- There is a **dependent relationship** between the variables, where the battery percentage depends on the charging time.

In a **dependent relationship** between two variables ***x*** and ***y***, where ***y* depends on *x***, we say:

- ***x*** is the **independent** variable.
- ***y*** is the **dependent** variable.

In this context, charging time is the independent variable, and battery percentage is the dependent variable.

CONCLUSIONS

- In the relationship analyzed in this activity, we saw that each value of time corresponds to exactly one battery percentage. This type of relationship is called a **function**.

A **function** is a **dependent relationship** between two variables, where each value of the independent variable corresponds **to exactly one value** of the dependent variable.

So, we say:

- The battery percentage **is a function of** time.

ACTIVITY 2

In the previous activity, we saw that the battery percentage of a phone **is a function of** charging time. This means there is a dependent relationship between these two variables, where each value of the independent variable (charging time) corresponds to exactly one value of the dependent variable (battery percentage).

ACTIVITY 2

Below are other examples of dependent relationships between two variables. Identify which of these are functions.

1

Independent variable: number of cartons (b)
Dependent variable: total number of eggs (C)
Note: each carton contains 12 eggs.

a) Is it possible that, for some value of the independent variable (number of cartons), there is more than one value of the dependent variable (total number of eggs)? Explain your answer.

b) Based on this, can we consider that the total number of eggs is a function of the number of cartons? explain your answer.

Work in pairs

ACTIVITY 2

2

Independent variable: type of fruit (*f*)
Dependent variable: color of the fruit (*C*)

a) Is it possible that, for some value of the independent variable (type of fruit), there is more than one value of the dependent variable (total fruit color)? Explain your answer.

b) Based on this, can we consider that the total number of eggs is a function of the number of cartons? explain your answer.

ACTIVITY 2

3

a) Can we consider the left thumbprint to be a function of the person? Explain your answer.

Independent variable: person (*p*)
Dependent variable: left thumbprint (*H*)

ACTIVITY 2

4

a) Can we consider the temperature to be a function of the time of day? Explain your answer.

Independent variable: time of day (h)
Dependent variable: temperature (T)

ACTIVITY 2

Whole class discussion

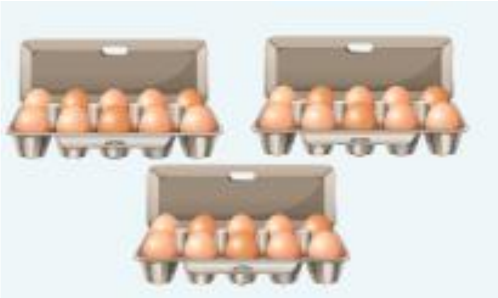
- What should you ask to decide if one of this situations there is a function? In the case of fruit type and color, should you check whether each color has a single fruit or whether each fruit has a single color?
- Can one value of the independent variable correspond to more than one value of the dependent variable here?
- In the case of temperature: Is there always one temperature per hour, or can it vary depending on other factors? Does it matter if the temperature is taken in the sun or the shade? Will the temperature be the same in two different towns at the same time?

CONCLUSIONS

Based on the analysis, we can identify three types of cases:

Relationships that are functions:

In some situations, it makes sense that each value of the independent variable has only one corresponding value of the dependent variable.



CONCLUSIONS

Based on the analysis, we can identify three types of cases:

Relationships that are not functions

Sometimes, it’s enough to find a case where one value of the independent variable is associated with more than one value of the dependent variable to say the relationship is not a function.

CONCLUSIONS

Based on the analysis, we can identify three types of cases:

Context-dependent cases

In some cases, whether or not the relationship is a function depends on the context. For example:

- Temperature T can be a function of the time of day h if measured in a small area, where we can assume one temperature per hour.

- But T would not be a function of h in a large area, like a region, where different places might have different temperatures at the same time.

FUNCTION NOTATION

To show that a dependent variable ***y*** is a function of the independent variable ***x***, we use the notation

$$y=f(x)$$

which is read as "**y** equals ***f*** of ***x***."

When we write ***y = f(x),*** the letter f indicates not only that ***y*** depends on ***x***, but also the relationship between the two variables. We don't always have to use the letter f to represent a function — we can use other letters like g, h, and so on.

FUNCTION NOTATION

Function	Notation	What does the function represent?
Battery percentage *P* is a function of charging time *t*.	$P = f(t)$	*f* represents the relationship between charging time and battery level.
Number of eggs *C* is a function of the number of trays *b*.	$C = g(b)$	*g* represents the relationship between the number of trays and total eggs.
A fingerprint *H* is a function of the person *p*.	$H = f(p)$	*f* represents the relationship between each person and their fingerprint.
Temperature *T* is a function of time *h*, under certain conditions.	$T = g(h)$	*g* represents the relationship between time of day and temperature.

ACTIVITY 3

Complete the following table using function notation *y=f(x)* for each case. When needed, choose letters to represent the variables and the function.

Description of the function	Function notation
f relates the maximum annual temperature (*T*) as a function of the year (*a*).	
g relates the perimeter of a circle (*P*) as a function of its radius (*r*).	
The area of the square *A* is a function of its side length *L*.	
Fatigue during exercise is a function of heart rate.	
The temperature of a cup of tea is a function of the time since it was served.	
The speed of an athlete is a function of elapsed time.	
Elapsed time is a function of an athlete's speed.	

ACTIVITY 3

Whole class discussion

- Did anyone use letters that were different from their classmate's? Can we use other letters?
- What are the initials of each variable? Is there a better word to describe the variable?
- Why is it important to use different letters for the functions even when the variables are the same?

CONCLUSIONS

- The letters used for variables and functions **can vary**, but what's most important is that **it's clear what each one stands for**.

- When writing a function with the notation ***y=f(x),*** it's important to use the correct order of letters: write the dependent variable first, then the function letter, and in parentheses the independent variable. For example, if ***f*** is the function that relates speed ***v*** to time ***t***, we write ***v=f(t),*** not the other way around.

LESSON SUMMARY

- We learned that one variable can depend on another—this means that the value of one changes when the other changes. This kind of connection is called a **dependency relationship**, where one variable is called the **dependent variable** and the other the **independent variable**.

- We found out that **a function** is a relationship where each value of the independent variable matches **exactly one value** of the dependent variable. In other words, there can't be two different dependent values for the same independent value.

LESSON SUMMARY

- We learned how to write functions using **the notation *y=f(x)***, where *y* is the dependent variable and *x* is the independent one. The letter *f* represents the relationship between them.

- We saw that even though we can use any letters to represent the variables and the function, it helps to use letters that remind us what they stand for—like the first letters of the words.

- We also saw that to avoid confusion, it's important to choose different letters when variables have similar names.

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