MATCON

MATEMÁTICA CONECTADA

TEACHER GUIDE

Lesson 1 - Functions Unit

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PLAN DE REACTIVACIÓN EDUCATIVA seamos Comunidad

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Lesson Overview

Lesson goal

Identify variables that change together in everyday contexts.

Lesson's role within the unit

This first lesson in the functions unit helps students understand what variables are and how they relate to one another, laying the groundwork for introducing the concept of a function.

Mathematical Actions

MA1. Identify characteristics that can change in everyday contexts.
MA2. Determine the possible values a changing characteristic can take.
MA3. Use the term variable to refer to characteristics or quantities that change.
MA4. Identify variables that change together.
MA5. Describe the dependency relationship between two variables that change together.

Lesson preview

This lesson consists of two activities. In Activity 1, students identify quantities or characteristics that can vary in everyday situations, determine the values these can take, and analyze the factors that cause them to vary. They also recognize situations in their environment where a quantity or characteristic does not change. From this activity, the notion of variable is introduced to describe quantities or characteristics that can change (MA1 and MA2).

In Activity 2, students identify pairs of related variables in different situations. They describe these relationships, determine the possible values the variables can take, and explain how one variable depends on the other. Through this activity, students are expected to recognize that some variables are related and change together (MA1, MA2, MA3, MA4, and MA5).

Activity 1

¿It changes?

1 **Think about the noise level in the classroom and, in pairs, answer the following questions:**

a. Is it a characteristic that can change?
b. What values can it take?
c. What do these values depend on?

View Expected Response

a). The noise level in the classroom is a characteristic that can change, as it may increase or decrease depending on various factors, such as the number of people speaking, the volume of their voices, or the presence of external sounds.

b). The values that the noise level can take depend on the type of scale being used, which may be qualitative, for example, "low," "moderate," "high", or numerical, such as a decibel (dB) scale.

c). The possible values of the noise level are influenced by many factors, such as:
- The number of people in the room
- The type of activity students are engaged in
- Noise coming from outside the classroom

2 In the table below, list characteristics from your everyday environment (such as your classroom, school, or home) that can change. Also write down the possible values they can take and what causes them to change. Use the temperature example in the table as reference.

Characteristic	Values	What causes it to have different values?
Temperature in the classroom	15°C, 20°C, etc ...	The time of day, whether the room is ventilated, etc.

View Expected Response

Characteristic	Values	What causes it to have different values?
Temperature in the classroom	15°C, 20°C, etc ...	The time of day, whether the room is ventilated, etc.
Number of students in the room	0,1,2,3	class period, what the class is doing at that moment, etc.
Level of cleanliness in the room	low, medium, high	time of day, what the class is doing at that moment, etc.
Light level in your house	low, medium, high	time of day, number of curtains closed, etc.

3 Finally, think about your classroom, school, or home and name characteristics that almost never change or are very unlikely to change. For example, the number of windows in the classroom.

View Expected Response

Number of doors in your home, color of the classroom walls, number of rooms in the school, etc.

Suggested Teacher Guidance – Activity 1

Objective
Identify characteristics in everyday contexts that may or may not change, determine their possible values, and the factors that cause them to vary.

Beginning of the lesson
Ask your students to observe the classroom and reflect on what things change throughout the day and what things stay the same. Explain that in this lesson they will explore characteristics of their environment that can change and others that remain constant.

First part

Work in Pairs

Present item 1 and organize students into pairs. Clarify that the values of the characteristics can be either numbers or qualities.

Whole class discussion

Ask students to share their responses and facilitate a discussion in which they:

- Recognize that the noise level in the classroom can change depending on how many students are present, how loudly they speak, and any sounds coming from outside.
- Explore ways to describe the values of the noise level using both qualitative scales (such as “low,” “moderate,” “high”) and numerical measures (How could we measure noise level using numbers?).
- Identify factors that influence the noise level, such as the number of people in the room or the type of activity taking place (What would make the noise increase or decrease in the classroom?).

Second part

Work in Pairs

Present item 2 and organize students into pairs. While monitoring, select a variety of responses, including qualitative scales, quantitative measures, and incorrect answers, to analyze during the whole-class discussion

Whole class discussion

Ask students to present their responses and guide a discussion in which they:

- Argue whether the identified characteristics can change or not (Can this characteristic take on different values? If so, which ones?).
- Analyze whether there are other ways to express the values of those characteristics (Can the values of these characteristics be expressed without numbers? Can they be represented using numbers?).
- Discuss whether the factors mentioned actually affect changes in the identified characteristics (How does that factor cause the characteristic to change?).

Third part

Individual work

Present item 3 and give students some time to think about characteristics in their environment that do not change.

Whole class discussion

Guide the discussion so that students recognize that, unlike variable characteristics, these characteristics maintain a single constant value (How many different values can the mentioned characteristics take? If we observe them at another time, would they have a different value?)

Conclusions

Summarize the key points from the discussions and connect students' responses to the following key ideas:

- In our environment, some characteristics can change while others remain constant.
- The values of variable characteristics can be represented as either qualities or numbers.

Defining key mathematical ideas

In mathematics, the following terms are used:

Variable: to describe characteristics that can change, such as the noise level, the number of people, or the level of cleanliness in a room.

Constant: to describe characteristics that do not change, such as the number of doors in a house, the color of the classroom, or the number of rooms in the school.

Anticipated Responses and Suggestions

- In item 1, some students may not understand what is meant by the values that the noise level can take. In that case, provide an alternative example, such as the speed of a car, which can be expressed numerically (0 km/h, 30 km/h, 60 km/h, etc.) or qualitatively ("slow," "moderate," "fast").

- If your students are not familiar with numerical scales used to measure noise, you can show them a mobile app that measures ambient sound in decibels.

Activity 2

What are the variables?

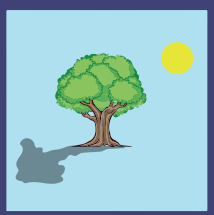
Look at the images on the right. For each one:

1. Identify two variables present in the situation depicted.
2. Describe the possible values that these variables can take.
3. Indicate whether one of the variables depends on the other.
4. Explain how that variable changes when the other variable changes.

You can use the coffee example as a reference.

Situation (example)				
	**Variables**	Coffee temperature	**Possible valuess**	10°C, 20°C,...
		Time since served		0, min, 5 min, 10 min,...
	One variable depends on the other?	The temperature of the coffee depends on how much time has passed since it was served.		
	How does that variable change when the other variable changes?	The temperature of the coffee decreases as time passes since it was served.		

Situation (example)				
	**Variables**		**Possible valuess**	
	One variable depends on the other?			
	How does that variable change when the other variable changes?			

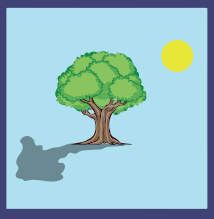
Situation (example)				
	**Variables**		**Possible valuess**	
	One variable depends on the other?			
	How does that variable change when the other variable changes?			

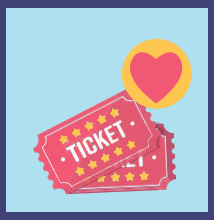
Situation (example)	Variables		Possible valuess	
		Number of movie tickets sold		
		Number of mentions of the movie on social media		
	One variable depends on the other?			
	How does that variable change when the other variable changes?			

View Expected Response

Varied responses depending on the characteristics that can be observed in the image.

Situation (example)	Variables		Possible valuess	
		Height of a tree		0 cm, 1 cm, 2 cm,...
		Amount of water per day		0 litros, 1 litro, 2 litros,...
	One variable depends on the other?	The height of the tree depends on the amount of water it receives each day.		
	How does that variable change when the other variable changes?	The tree's height increases as it receives more water daily, and its growth decreases if it receives less water.		

Situation (example)	Variables		Possible valuess	
		Length of the tree's shadow		0 m, 0.5 m, 1 m, 5 m...
		Position of the sun		near the horizon, above the tree, etc.
	One variable depends on the other?	The length of the tree's shadow depends on the position of the sun.		
	How does that variable change when the other variable changes?	The length of the shadow increases as the sun gets closer to the horizon and decreases when the sun is directly overhead.		

Situation (example)	Variables	Number of movie tickets sold	Possible valuess	1000, 2000, 3000, etc.
		Number of mentions of the movie on social media		100, 200, 300, etc.
	One variable depends on the other?	Both variables may depend on each other.		
	How does that variable change when the other variable changes?	More ticket sales can lead to more social media mentions, while an increase in mentions might attract more people to watch the movie, increasing ticket sales.		

Suggested Teacher Guidance – Activity 2

Objective
Identify related variables in different situations by analyzing the dependency between them and describing how a change in one variable affects the other.

Work in pairs

Explain the activity, organize students into pairs, and monitor their work, selecting a variety of responses to discuss during the whole-class sharing.

Whole class discussion

Ask students to share their responses and guide a discussion in which they:

- Evaluate whether the identified variables make sense in the context of the image (How is that variable represented in the image?).
- Justify that the identified characteristics are variables (How do we know that this characteristic can change?).
- Discuss the possible values of the variables (What other values could this variable take? Can we express them in another way, such as with numbers or words?).
- Identify and explain whether one variable depends on the other (Would this variable change if the other one didn't?).
- Analyze how one variable changes when the other changes (What happens to this variable if the other increases or decreases?).

Conclusions

Summarize the key points from the discussions and connect students' responses to the following idea:

- Some variables are related in such a way that a change in one causes a change in the other. In this case, we say that one variable depends on the other.

Defining key mathematical ideas

If one variable depends on another, we say there is a dependency relationship between them.

In the situations analyzed, we identified examples of these dependency relationships:

- The height of a tree depends on the amount of water it receives each day (as long as the amount is not excessive).
- The length of a tree's shadow varies depending on the position of the sun.

A simple way to explain how one variable changes in relation to another is to describe what happens to the first variable when the other one changes. For example:

- If the tree receives more water, it grows more; if it receives less water, it grows less.
- When the sun is near the horizon, the tree's shadow is longer; when the sun is directly above the tree, the shadow is shorter.

- There are situations in which it is not clear which variable depends on the other. For example, in the case of movie tickets sold and social media mentions, both can influence each other, making it difficult to determine which one causes the change in the other.

Anticipated Responses and Suggestions

- Some students may have difficulty assigning values to certain variables. Encourage them to think about which aspects are changing and to describe those changes using words or numbers when possible.

- It may also happen that they identify two related variables but are unable to recognize which one depends on the other. In those cases, present simple examples. For instance: the height of a plant depends on the amount of sunlight it receives, but sunlight does not change when the plant grows taller.

- In situations where both variables may influence each other, students might reject the idea of arbitrarily considering one as dependent on the other. Explain that even if there is no natural dependency, it is still valid to establish one arbitrarily in such cases.

Exit slip

Look at the following image showing a person climbing San Cristóbal Hill.

Which of the following options correctly represents two related variables in this context, indicating which one depends on the other and how they are related?

a)
- Variables: Height of San Cristóbal Hill and the amount of water the person drinks on the way to the top.
- Relationship: The amount of water consumed depends on the height of the hill.

b)
- Variables: Distance traveled by the person and elapsed time.
- Relationship: The distance traveled increases as time passes.

c)
- Variables: Heart rate and the weather during the run.
- Relationship: The runner's heart rate increases on sunny days and decreases on cloudy days.

Check possible understandings

Assessment Indicator	Identify variables and describe their dependency relationship (i.e., how one variable changes in relation to the other) in a contextualized situation.

Choice	Correct answer	Possible understandings behind the error
a		They do not recognize that the height of the hill is a constant value, and based on that, they consider a relationship between the amount of water and the height to be plausible.
b	X	
c		Although heart rate and weather are plausible variables in this context, the proposed relationship between them is not. Students who choose this option are likely focusing only on identifying variables, without critically evaluating the relationship between them.

Lesson Summary

Lesson 1 - Functions Unit

In this lesson:

- We learned to use the term **variable** to refer to characteristics that can change, and the word constant to describe those that remain the same.

Examples of variables:	Examples of constants:
• The noise level in the classroom • The number of people in a room • The level of cleanliness in the classroom	• The number of doors in your home. • The color of the classroom walls. • The number of rooms in the school.

- We learned that variables can take on **numerical values** or **qualitative values.**
  - **Numerical variable**: Number of people in the classroom. Its values are numbers like 0, 1, 2, 3, etc.
  - **Qualitative variable**: Level of cleanliness in the classroom. Its values can be qualities like “low,” “medium,” or “high.”

- We discovered that in some cases, a change in one variable depends on a change in another.
  - The temperature of the coffee depends on how much time has passed since it was served.
  - The height of a tree depends on the amount of water it receives each day.
  - The length of a tree’s shadow depends on the position of the sun.

- We learned to describe in words how one variable changes as the other changes:
  - If the tree receives more water, it grows more; if it receives less, it grows less.
  - When the sun is near the horizon, the tree’s shadow is longer; when the sun is directly overhead, the shadow is shorter.

- We also discuss that in some situations, it **is not clear which variable depends on the other,** since both may influence each other—for example, movie ticket sales and social media mentions.

Mathematical Terms I Can Now Use

- variable
- constant
- dependency between variables