MAT CON

MATEMÁTICA CONECTADA

TEACHER GUIDE

Lesson 2 - Function Unit

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LESSON OVERVIEW

Lesson Goal

Identify whether a relationship between variables is a function and represent it using the notation $y=f(x)$.

Lesson's role within the unit

Lesson 2 follows the introduction of the concept of variables and the analysis of dependent relationships between two variables. In this lesson, the concept of a function is introduced using the idea of a correspondence between variables. Other interpretations will be explored later in the unit.

Lesson 1
Introducing Variables

Lesson 2
Function Concept

Lección 3
Tabular and Graphical Representation of a Function

Mathematical Actions

MA1. Analyze, considering the context and the nature of the variables, whether each value of the independent variable corresponds to exactly one value of the dependent variable.

MA2. Determine whether a given relationship meets the definition of a function.

MA3. Represent the functional relationship between two variables using the notation $y = f(x)$.

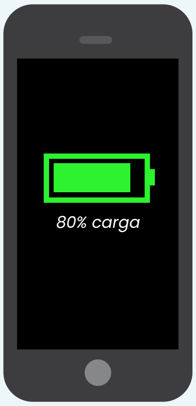
Lesson Preview

In Activity 1, building on the context from the warm up activity, students identify which variable depends on the other and analyze whether it's possible for a phone to reach more than one battery level at the same time after being charged for a certain period (T1). This activity introduces the terms “independent variable” and “dependent variable” and uses them to introduce the definition of a function.

In Activity 2, students are given situations with a given independent and dependent variable. They must explain, based on the nature of the variables, whether each value of the independent variable leads to only one value of the dependent variable, or if several values are possible (T1). Through this analysis, they recognize in which cases the relationship meets the definition of a function (T2). During the class discussion, the notation $y = f(x)$ is introduced to represent the functions related to the variables in each case.

In Activity 3, students write functions using the notation $y = f(x)$ (T3). They choose letters to represent the variables and the function, noting that there may be several reasonable options. In the class discussion, they compare their choices and agree on general

WARM UP | A phone's battery



Think about your phone's battery level, as a percentage, and the amount of time it has been charging.

1. What are the variables involved in this situation?
2. What values can these variables take?
3. What happens to one variable when the other increases?

View expected response

1. The variables involved are the phone's battery percentage and the charging time.
2. The battery percentage can range from 0% to 100%, while the charging time can be any positive number.
3. As time passes, the phone's battery percentage increases.

SUGGESTED TEACHER GUIDANCE – WARM UP

Beginning of the lesson

Tell students that in this class they will explore a type of relationship between variables called a function, starting with an activity that will help them recall key concepts for the lesson.

Guiding the warm up

Present the activity and have various students share their answers. Guide the discussion, making sure they:

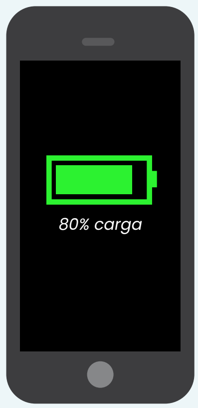
- Identify the variables in the situation (Do you remember what a variable is? Why are the phone's battery percentage and charging time considered variables?).
- Discuss the possible values of the variables (How is battery percentage measured? How is charging time measured?).
- Explain how one variable changes when the other changes (What happens to the battery percentage as charging time increases?).

What we should remember for this lesson

Connect what students answered during the activity with the following mathematical ideas:

- A variable is a feature that can change. In this case, the variables are the battery percentage and the charging time.
- In general, variables can take on numerical values or qualities. In this case, we consider both variables to be numerical: battery percentage ranges from 0% to 100%, while charging time can be measured in minutes.
- We can describe the relationship between variables by explaining how one changes when the other changes. For example, as time passes, the phone's battery percenta-

ACTIVITY 1 | Can there be more than one battery percentage at a time?



Let's go back to the earlier context, where we looked at the relationship between your phone's battery percentage and the time since it started charging.

View expected response

1. In this situation, we identified two variables. Which variable depends on the other?

The battery percentage depends on the charging time.

2. Suppose your phone was completely dead and you charge it for 20 minutes. Is it possible that at that moment the battery percentage is both 50% and 55%.

No, at a given moment there can only be one battery percentage. If it shows 50% or 55% at 20 minutes, it will be one or the other, but not both at the same time.

3. In general, if you charge your phone for a set amount of time, is it possible for it to show more than one battery percentage at that same moment?

No, at any given moment, the battery percentage is unique. The phone has only one level of charge at a time.

SUGGESTED TEACHER GUIDANCE – ACTIVITY 1

Objective

Analyze the dependent relationship between two variables in a specific context, recognizing that in this relationship, each value of the independent variable corresponds to exactly one value of the dependent variable.

Work in pairs

Explain the activity and organize students into pairs to discuss and answer the questions.

Whole class discussion

Lead a discussion where students share their answers and reasoning. Make sure they:

- Recognize that the battery percentage depends on the charging time (What feels more natural in this context: for the percentage to depend on time, or for time to depend on the percentage?).
- Notice that for each amount of charging time, there can only be one battery level (Is it possible for the phone to show two different battery percentages at the same time? Why?).

Defining key mathematical ideas

Summarize the key points from the discussion and connect students' responses to the following mathematical ideas:

- There is a dependent relationship between the variables, where the battery percentage depends on the charging time.

In a dependent relationship between two variables x and y, where y depends on x, we say:

- x is the independent variable.
- y is the dependent variable.

In this context, charging time is the independent variable, and battery percentage is the dependent variable.

- In the relationship analyzed in this activity, we saw that each value of time corresponds to exactly one battery percentage. This type of relationship is called a function.

A function is a dependent relationship between two variables, where each value of the independent variable corresponds to exactly one value of the dependent variable.

So, we say:

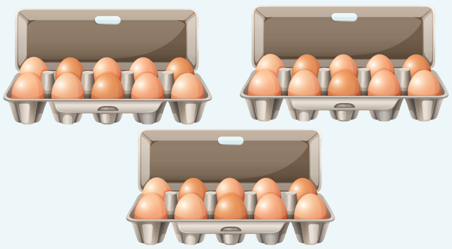
- The battery percentage is a function of time.
- The battery percentage depends on time.

Anticipated responses and suggestions

- Some students might interpret the dependency the other way around, thinking of time as a function of battery percentage. While this interpretation can make sense in certain cases (like figuring out how long it takes to reach a specific charge level), the focus of this activity is to analyze the function that describes how battery percentage changes with time.

ACTIVITY 2 | Is it a function?

In the previous activity, we saw that the battery percentage of a phone is a function of charging time. This means there is a dependent relationship between these two variables, where each value of the independent variable (charging time) corresponds to exactly one value of the dependent variable (battery percentage). Below are other examples of dependent relationships between two variables. Identify which of these are functions.



Situation 1
Independent variable: number of cartons (b)
Dependent variable: total number of eggs (C)
Note: each carton contains 12 eggs.

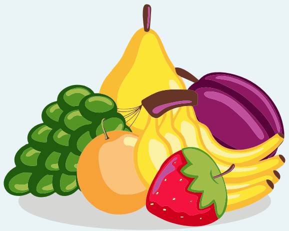
a) Is it possible that, for some value of the independent variable (number of cartons), there is more than one value of the dependent variable (total number of eggs)? Explain your answer.

b) Based on this, can we consider that the total number of eggs is a function of the number of cartons? explain your answer.

View expected response

a) The note specifies that each carton contains the same number of eggs (12), so for each value of the number of cartons b (independent variable), there is a single value for the total number of eggs C (dependent variable).

b) This means we can consider the total number of eggs C as a function of the number of cartons b.



Situation 2
Independent variable: type of fruit (f)
Dependent variable: color of the fruit (C)

a) Is it possible that, for some value of the independent variable (type of fruit), there is more than one value of the dependent variable (fruit color)? Explain your answer.

b) Based on this, can we consider that the color of the fruit is a function of the type of fruit? Explain.

View expected response

a) Some types of fruit can come in more than one color—for example, apples can be green or red, and watermelons can have different colors on their surface.

b) According to the definition of a function, the color of the fruit is not a function of the type of fruit, since a single type of fruit can have more than one color.

Situation 3
Independent variable: person (p)
Dependent variable: left thumbprint (H)

a) Can we consider the left thumbprint to be a function of the person? Explain your answer.

View expected response

Each person has a unique left thumbprint. That is, for each value of the independent variable p (person), there is exactly one value of the dependent variable H (thumbprint). Therefore, the left thumbprint is a function of the person.

Situation 4
Independent variable: time of day (h)
Dependent variable: temperature (T)

Can we consider the temperature to be a function of the time of day? Explain your answer.

View expected response

The answer depends on where the temperature is measured. If it's measured in a small area, like a town or a specific location like a city square, then it makes sense to say that for each hour of the day h, there is a single temperature T.

However, if the location is large, like a region, then there could be several different temperatures at the same time of day, for example at the coast, in the valleys, or near the mountains. Also, at the same time, temperature can differ depending on whether it's

SUGGESTED TEACHER GUIDANCE – ACTIVITY 2

Objective

Identify whether a relationship between two variables can be considered a function, using the definition of a function and analyzing the nature of the variables in specific situations.

Work in pairs

Explain the task and highlight that students must check if the variables in each case meet the definition of a function. Monitor their work and choose a variety of responses to discuss with the whole class.

Whole class discussion

Lead a discussion where students share their answers and reasoning. Make sure they understand that, to determine whether a relationship is a function, they need to consider:

- Asking the right question: (What should they ask to decide if it's a function? In the case of fruit type and color, should they check whether each color has a single fruit or whether each fruit has a single color?)
- Context analysis: (Can one value of the independent variable correspond to more than one value of the dependent variable here? In the case of temperature: Is there always one temperature per hour, or can it vary depending on other factors? Does it matter if the temperature is taken in the sun or the shade? Will the temperature be the same in two different towns at the same time?)

Defining key mathematical ideas

Summarize the main ideas from the discussion and connect them to students' responses:

- Based on the analysis, we can identify three types of cases:
  1. Relationships that are functions:

  In some situations, it makes sense that each value of the independent variable has only one corresponding value of the dependent variable. For example:

  - The total number of eggs C is a function of the number of trays b, since each number of trays gives one specific number of eggs.

- The fingerprint of the left thumb *H* is a function of the person *p*, assuming each person has a unique fingerprint.

2. Relationships that are not functions:

Sometimes, it's enough to find a case where one value of the independent variable is associated with more than one value of the dependent variable to say the relationship is not a function. For example:

- The color of a fruit C is not a function of the type of fruit f, since one type of fruit (like an apple) can come in different colors.

3. Context-dependent cases: :

In some cases, whether or not the relationship is a function depends on the context. For example:

- Temperature *T* can be a function of the time of day h if measured in a small area, where we can assume one temperature per hour.
- But *T* would not be a function of h in a large area, like a region, where different places might have different temperatures at the same time.

- To express concisely that one variable is a function of another, we use the following notation:

Function Notation

To show that a dependent variable y is a function of the independent variable x, we use the notation

$$y=f(x)$$

which is read as "y equals f of x."

When we write $y = f(x)$, the letter f indicates not only that y depends on x, but also the relationship between the two variables. We don't always have to use the letter f to represent a function — we can use other letters like g, h, and so on. For example:s:

Function	Notation	What does the function represent?
Battery percentage *P* is a function of charging time *t*.	$P=f(t)$	*f* represents the relationship between charging time and battery level.
Number of eggs *C* is a function of the number of trays *b*.	$C=g(b)$	*g* represents the relationship between the number of trays and total eggs.
A fingerprint *H* is a function of the person *p*.	$H = f(p)$	*f* represents the relationship between each person and their fingerprint.
Temperature *T* is a function of time *h*, under certain conditions.	$T = g(h)$	*g* represents the relationship between time of day and temperature.

Anticipated responses and suggestions

It's important that students explain their answers. It's not enough to simply say "yes, it's a function" or "no, it's not a function", they need to explain the reasoning behind their response.

Emphasize how context plays a key role in the analysis. Ask guiding questions like:

- What assumptions are you making about this situation?
- Would the answer change if the context were different?
- What if the boxes weren't full.
- How does the way we measure temperature affect the result

ACTIVITY 3 | Function notation

Complete the following table using function notation $y=f(x)$ for each case. When needed, choose letters to represent the variables and the function.

Description of the function	Function notation
f relates the maximum annual temperature (T) as a function of the year (a).	
g relates the perimeter of a circle (P) as a function of its radius (r).	
The area of the square A is a function of its side length L.	
Fatigue during exercise is a function of heart rate.	
The temperature of a cup of tea is a function of the time since it was served.	
The speed of an athlete is a function of elapsed time.	
Elapsed time is a function of an athlete's speed.	

View expected response

Description of the function	Function notation
f relates the maximum annual temperature (T) as a function of the year (a).	$T=f(a)$
g relates the perimeter of a circle (P) as a function of its radius (r).	$P=g(r)$
The area of the square A is a function of its side length L.	Various answers for the function letter, for example: $A=s(L)$
Fatigue during exercise is a function of heart rate.	Various answers for the letters of the function and variables, for example: $C=m(f)$
The temperature of a cup of tea is a function of the time since it was served.	Various answers for the letters of the function and variables, for example: $T=h(t)$
The speed of an athlete is a function of elapsed time.	Various answers for the letters of the function and variables, for example: $v=f(t)$
Elapsed time is a function of an athlete's speed.	Various answers for the letters of the function and variables, for example: $t=g(v)$

SUGGESTED TEACHER GUIDANCE – ACTIVITY 3

Objective

rite functions using the notation $y=f(x)$, choosing appropriate letters for the variables and the function in different contexts.

Independent work

Make sure students clearly understand the task and monitor their progress. Pay special attention to the following:

- **Choosing letters for variables and functions**: Check if students are picking letters that make sense in the context or if they're just using default ones like x, y, and f.

- **Possible difficulties**:

  - *Fatigue and heart rate*: Some students might use f for fatigue, which could cause confusion if they also use f for the function. Make sure they know they need different letters for the variables and the function.
  - *Temperature and time*: Both start with the letter "*t*". Remind students they need to clearly distinguish between them. They could use uppercase and lowercase letters (T for temperature and t for time), or different letters altogether.
  - *Speed and time*: In the last two cases, the variables are the same, but the functions are different. Make sure students understand that they need different letters for each function, even if the variables stay the same.

- **Correct use of function notation**: Check that students follow the correct order:
  dependent variable = function(independent variable).

Whole class discussion

Ask students to share examples you observed while monitoring. During the discussion, make sure they:

- Understand there's no single "correct" way to choose letters. Different answers are fine, as long as it's clear which letter represents which variable. (Did anyone use letters that were different from their classmate's? Can we use other letters?)

- See the benefit of using the first letter of the variable's name, like v for speed or d for distance. (What are the initials of each variable? Is there a better word to describe the variable?)

- Discuss the challenges of working with variables and functions that start with the same letter (like f for both "fatigue" and "function"), and the importance of picking distinct letters, either by using different ones or using uppercase and lowercase (What strategy did we use to tell them apart?).

- Notice that the last two situations have the same variables but represent different functions. In these cases, the variables can stay the same, but the functions need different letters (Why is it important to use different letters for the functions even when the variables are the same?).

Defining key mathematical ideas

Summarize the key ideas from the discussion and link them to these points:

- The letters used for variables and functions can vary, but what's most important is that it's clear what each one stands for.

- Even though we can use any letter, it's helpful to use the initial letter of the variable's name—like v for speed or d for distance—because it makes them easier to remember.

- If two variables start with the same letter (like time and temperature), we need to pick distinct letters to avoid confusion. We can:

    - Use different letters.
    - Use the same letter in lowercase and uppercase, like t for time and T for temperature.

  When writing a function with the notation $y=f(x)$, it's important to use the correct order of letters: write the dependent variable first, then the function letter, and in parentheses the independent variable. For example, if f is the function that relates speed v to time t, we write $v=f(t)$, not the other way around.

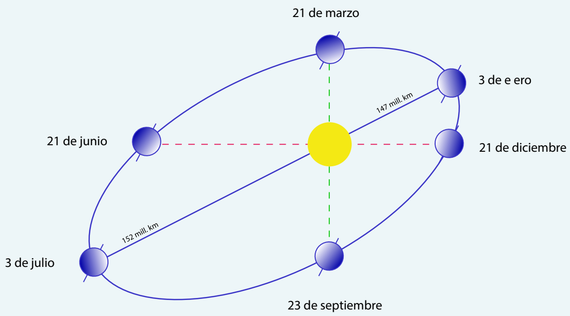
  Even though we're free to choose the letters, some are commonly used for certain variables. For instance, t for time, T for temperature, r or R for the radius of a circle, etc. Using these conventions helps everyone quickly understand what each letter represents.

Anticipated responses and suggestions

- If students start debating whether the situations are functions or not, remind them that the task assumes all the given relationships *are* functions.
- Some students might feel unsure about picking letters freely and prefer only using initials. Remind them that function notation is flexible, as long as the use of letters is clear and consistent.

EXIT TICKET

The following image shows Earth's position in its orbit around the Sun at different times of the year. A scientist claims that "the Earth's position in its orbit is a function of the day of the year."



Write a function that relates the Earth's position in its orbit to the day of the year using the notation $y=f(x)$.

Explain why you choose those letters.

Check possible understandings

Assessment Indicator	Identify dependent and independent variables in real world situations.

Answers may vary depending on the choice of letters for the variables and the function.

$P = f(d)$
P for "position" and *d* for "day".

It is important that the notation students use reflects the relationship—in this case, that position P depends on the day of the year d. Expressions such as P = d or P = f = d may indicate that students did not properly incorporate the notation introduced in the lesson. These responses could suggest that students see both quantities as equal rather than one depending on the other, or that they are unsure how to use function notation correctly.

LESSON SUMMARY

In this lesson:

- We learned that one variable can **depend on** another—this means that the value of one changes when the other changes. This kind of connection is called a **dependency relationship**, where one variable is called the **dependent variable** and the other the **independent variable**. For example:
    - The battery charge percentage P depends on the charging time *t*. As charging time increases, the charge percentage goes up.
    - In this case, *P* is the dependent variable and *t* is the independent one.
- We found out that a **function** is a relationship where each value of the independent variable matches **exactly one value** of the dependent variable. In other words, there can't be two different dependent values for the same independent value. For example:
    - The battery charge percentage *P* is a **function** of the charging time *t*, because for each amount of charging time, there's only one possible charge percentage. The
    - color of a fruit *C* **is not a function** of the fruit type *f*, because a fruit like an apple can come in different colors.
- We learned that in some situations, deciding whether a relationship is a function **depends on the context**. For example:
    - Temperature *T* **might be a function** of the time of day *h* if it's measured in a small space like a room—then we can assume there's only one temperature at each hour.
    - But if we measure temperature *T* across a large region, the relationship **wouldn't be a function** because the same hour *h* could have different temperatures in different places.
- We learned how to write functions using the **notation y=f(x)**, where y is the dependent variable and x is the independent one. The letter f represents the relationship between them. For example:
    - If f is the function that relates battery charge *P* to charging time *t*, then we write $P=f(t)$.
      This is read as "*P* equals *f* of *t*."
    - We can also say that *P* depends on *t*, or that *P* is a function of *t*.

- We saw that even though we can use any letters to represent the variables and the function, it helps to use letters that remind us what they stand for—like the first letters of the words. For example:
    - We use P for the perimeter of a circle and r for its radius.
    - If g is the function that relates perimeter and radius, we write P=g(r).
- We also saw that to avoid confusion, it's important to choose different letters when variables have similar names. For example:
    - We use t for time and T for temperature, so they don't get mixed up.

Mathematical Terms I Can Now Use

- dependent and independent variable
- function
- function notation, $y = f(x)$