MATCON

MATEMÁTICA CONECTADA

TEACHER GUIDE

Lesson 2 - Probability Unit

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Lesson Overview

Lesson goal

Assign qualitative likelihood levels to different observations of a random situation, using the categories: impossible, possible, or certain.

Lesson's role within the unit

This lesson is the second in the unit and introduces the concept of likelihood levels using qualitative language.

Mathematical Actions

MA1. Assess the likelihood level of an observation from a random situation in an intuitive way.
MA2. Assess the level of likelihood of an observation resulting from a random situation using the terms "impossible," "likely," and "certain."
MA3. Recognize the subjectivity involved in assessing the likelihood of an observation resulting from a random situation.
MA4. Objectively assess the likelihood of an observation in a random situation when all relevant information is known.

Lesson preview

In this lesson, students will explore how to assess the likelihood of observations from random situations. They will move from intuitive judgments to classifying observations as "impossible," "possible," or "certain," using a qualitative scale of likelihood levels.

In Activity 1, students will analyze everyday examples by comparing the likelihood of different observations in an intuitive way (MA1). This work sets the stage for introducing the formal language that will be used in Activity 2, where observations are classified according to their **level of likelihood** (MA2).

Finally, in Activity 3, students will work with situations where more information is available, allowing them to assess the likelihood level of in a more objective way. Through these experiences, they will be encouraged to reflect on the subjectivity that often accompanies the assessment of random events and to consolidate key ideas about the **nature of chance** (MA3, MA4).

Warm up

Random Situations

The following situations are described below:

1. Go out to the schoolyard and measure the noise level at school.
2. Record waiting time at school snack bar.
3. Go out to the schoolyard and count the number of people in the schoolyard.
4. Record how long it takes for the schoolyard to clear out after recess.

Which of these situations are random? Why?

View Expected Response

1. The noise level cannot be predicted with certainty, as it depends on multiple varying factors (such as the number of people, the activities they are doing, weather conditions, etc.). Therefore, this situation is random.
2.
3. The waiting time varies depending on how many students arrive, what time they arrive, and how quickly they are served, which makes it difficult to predict. Therefore, this situation is also random.
4.
5. The number of people in the schoolyard can vary depending on factors such as the time of day, the weather, or school events. Therefore, it cannot be predicted with certainty before observing.
6.
7. This time can change depending on student behavior, the presence of teachers, or other unpredictable factors. Since its duration cannot be anticipated, we say that this situation is also random.

Suggested Teacher Guidance – WARM UP

Beginning of the lesson

Tell your students that in this lesson they **will explore how likely it is for a certain outcome or observation to occur in a random situation**. For example: How likely is it to rain tomorrow? To begin, offer a brief activity that helps them recall what we understand by a random situation.

Guiding the Warm Up

Present the activity and invite different students to share their answers to the questions in a whole-class discussion. Allow space for disagreement if it arises. During the discussion, aim for students to:

- Justify whether or not a situation is random based on whether the observation can be predicted or anticipated (Can we predict how long someone will wait at the school snack bar? Can you be certain how many people will be in the schoolyard before going out to observe?)

What we should remember for this Lesson

Use students' responses in this activity to reconnect with the mathematical terms and ideas from the previous lesson:

- There are situations in which we cannot anticipate what will be observed due to uncertainty. We say these situations are random, and that chance is involved.

Activity 1

In pairs, consider the following situations with three possible associated observations and answer the questions.

	Possible observations		
Random Situation	**Observation 1**	**Observation 2**	**Observation 3**
Measure the noise level at school.	Noise level is very loud during recess.	Noise level is null during recess.	Noise level is louder during recess than during class.
Record waiting time at school snack bar.	Waiting time is less than a couple of minutes.	Waiting time is more than an hour.	Waiting time is longer during recess than during class.
Counting the number of people in the schoolyard.	There's no one in the schoolyard during recess.	A few people are there during PE class.	The schoolyard is more crowded during recess than during class.
Recording how long it takes for the schoolyard to clear out after recess.	The schoolyard is empty right after the bell rings.	The schoolyard takes less than 5 minutes to empty.	The schoolyard takes less than half an hour to empty.

1. For each situation:

A How likely do you think each observation is?

View Expected Response

- In the first situation, observations 1 and 3 seem likely, while observation 2 is practically impossible.
- In the second situation, observations 1 and 3 seem likely, while observation 2 is impossible.
- In the third situation, observation 1 is impossible, observation 2 is likely, and observation 3 is certain.
- In the fourth situation, all of them could happen, although 1 and 3 seem less likely to occur.

B Would you say that some observations are more likely than others?

View Expected Response

Yes, for example, observation 3 in the first situation is something that is almost certain to happen, while observation 1 is less likely and observation 2 is impossible.
Another example: in the third situation, observation 1 is impossible because if it's recess, there will be people in the schoolyard. Observation 3 always happens — there are more people in the yard during recess than during class time.

Suggested Teacher Guidance – Activity 1

Objective
Intuitively assess the likelihood level of a specific observation associated with a random situation.

Work in Pairs

Present the situations in the table and organize students into pairs to discuss item 1. While monitoring, invite them to explain why they think some observations seem more or less likely than others in each case. Although it is not necessary for students to use formal language at this stage, pay attention to moments when they use terms like "impossible" or "certain" (or similar expressions), as these will be useful for the discussion that follows.

Whole class discussion

Facilitate a whole-class discussion so students can share their answers to the questions in item 1. Make sure they:

- Discuss and reflect on the terms they used to describe the likelihood of each observation (What words did you use to talk about situations you considered very unlikely?).
- Work toward consensus on which observations they consider "more likely" than others within each situation (Why do you say one observation is more likely than another? What are you basing that on?).
- Recognize that both the available information and their personal experience help them assess and compare how likely an observation is (What made you think that situation was "less likely" than the one another group suggested?).

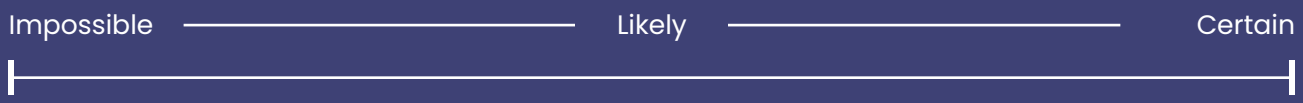
Conclusions

Summarize the key points from the discussions and connect students' responses to the following ideas:

- In random situations, observations can have different levels of likelihood.
- To describe these levels, we will consider an observation that never occurs as impossible; one that always occurs as certain; and one that may occur, but doesn't always, as likely.
- The presence of likely observations is what gives a situation its random nature.

Defining key mathematical ideas

The levels of likelihood can be represented in a diagram like the one below, where observations that are more likely to occur are placed closer to Certain, and those that are less likely are placed closer to Impossible.



Anticipated Responses and Suggestions

In this first activity, we use the word **observation** to refer to an event within a random experiment. For example, when we observe "The schoolyard becomes empty as soon as the bell rings" and "The schoolyard takes less than 5 minutes to become empty," we are referring to two possible events associated with the experiment "Measuring the time it takes for the schoolyard to become empty after the bell rings following recess". The focus is on comparing how likely each event is to occur, rather than on the specific language used to describe them, since these terms will be formalized later on.

In the second question, where students are asked to compare levels of likelihood, some may argue that no observation is "more likely" than another, since in random situations, "all observations have the same level of possibility". In that case, encourage students to use the available information from each situation, as well as their own experience, to assess and compare which observations seem "more likely" than others. While all observations may be possible within a given situation, we can still **assess and compare their likelihood level based on our own reasoning and criteria.**

Activity 2

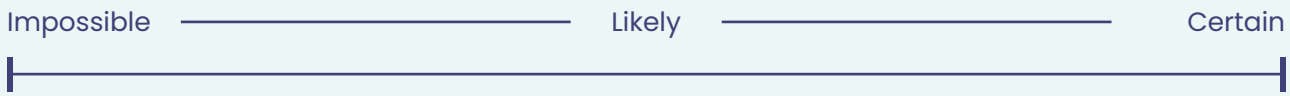
1 **In groups, classify the following observations according to their level of likelihood as "Impossible," "Possible," or "Certain".**

a. It will be cloudy tomorrow morning.
b. It rains in Puerto Montt on a winter day.
c. Toast with jam falls jam-side down.
d. A cat lands on its feet.
e. I won't tell anyone the gossip I just heard.
f. The sun rises in the morning.
g. The moon falls to Earth.
h. A rainbow appears in the sky after the rain.
i. I win five rounds of rock-paper-scissors in a row.
j. Chile qualifies for the next men's World Cup.
k. Alexis Sánchez wins an Olympic medal in archery.
l. I'll get an A on the next math test.

View Expected Response

a. It will be cloudy tomorrow morning. **Likely**
b. It rains in Puerto Montt on a winter day. **Likely**
c. Toast with jam falls jam-side down. **Likely**
d. A cat lands on its feet. **Likely**
e. I won't tell anyone the gossip I just heard. **Impossible**
f. The sun rises in the morning. **Certain**
g. The moon falls to Earth. **Impossible**
h. A rainbow appears in the sky after the rain. **Likely**
i. I win five rounds of rock-paper-scissors in a row. **Likely / Impossible**
j. Chile qualifies for the next men's World Cup. **Likely / Impossible**
k. Alexis Sánchez wins an Olympic medal in archery. **Impossible**
l. I'll get an A on the next math test. **Likely**

2 **Place the previous observations on the diagram below.**
Note: The closer an observation is to "Certain," the more likely it is to happen. On the other hand, the closer it is to "Impossible," the less likely it is to happen.



View Expected Response

Varied responses. It is expected that different groups will have different judgments. For example, for observation (n) "I'll get an A on the next math test," some groups may place it closer to "Impossible," while others may place it closer to "Certain."

(e) (j) (a) (i) (c) (h) (l) (b) (d) (f)
(g)
(k)

3 **Compare the diagrams from each group and reflect on the following questions:**

a. Are there differences in the level of likelihood assigned to each observation?
b. Why do you think this happens?

View Expected Response

Varied responses. The arguments should be based on differences in students' perceptions. For example, for the first observation, it depends on whether they are thinking of a winter day or a summer day.

Suggested Teacher Guidance – Activity 2

Objective
Assess the likelihood of a specific observation associated with a random situation using a scale with the terms “impossible,” “likely,” and “certain.”

Group Work

Present the list of observations and organize students into groups to discuss items 1 and 2. While monitoring, make sure the groups reflect on and discuss the level of likelihood they assign to each observation, so they can explain the reasoning behind their choices. This will help them place each situation along the likelihood scale.

Whole-Class Sharing

Organize students into the same groups as before to discuss item 3. Ask one representative from each group to present their group’s work to the rest of the class. Record at least two different diagrams on the board, and facilitate a discussion in which students:

- Notice that there are shared patterns (in the “certain” and “impossible” events), but that most of the variation appears in how the category “likely” is positioned on the scale.
- Explain that the differences between the diagrams are due to differences in individual perceptions. (For example: For “I’ll get an A on the next math test,” why does the placement vary?)

Conclusions

Summarize the key points from the discussions and connect students' responses to the following key ideas:

- "Certain" and "impossible" observations tend to be consistent, but the greatest variability in likelihood appears with "likely" observations.
- The assessment of the likelihood level can vary depending on each person's perceptions.

Anticipated Responses and Suggestions

Students may have difficulty distinguishing between something that is very likely and something that is certain, or between something that is very unlikely and something that is impossible. In that case, explore the difference between observations that are known with certainty and those that involve some degree of uncertainty. For example, "the sun rises in the morning" is certain, as it is governed by physical laws, whereas "it will be cloudy tomorrow morning" depends on the season or the location where the observation is made.

It is also possible that groups may struggle to reach consensus on how likely certain observations are. In that case, guide the discussion toward the reasoning behind their judgments and the role that subjectivity plays in assigning levels of likelihood.

Activity 3

In groups, imagine the following situation:

When you arrive at a birthday party, you receive a surprise candy bag. The birthday girl put them together in a bit of a hurry, so the bags are quite different from one another. When you receive your bag, you must take out one candy at random (without looking) and observe its color.

A For each of the following bags, indicate how likely it is that the candy you draw will be red. Justify your answer in each case

View Expected Response

It's likely to draw a red candy from the bag because there are candies of that color along with others.	**It's impossible** to draw a red candy from the bag because no red candies are inside.	**It's likely** to draw a red candy from the bag, because there are candies of that color along with others.	**It's certain** that a red candy will be drawn from the bag, because all the candies are red.	**It's impossible** to draw a red candy from the bag because it is empty, so there are no red candies to draw.

B How does chance play a role in this situation?

View Expected Response

There are three situations where chance does not play a role, because they involve impossible events (a bag with only blue candies or an empty bag) or a certain event (a bag with only red candies). In contrast, in bags 1 and 3, chance is involved because there are candies of different colors.

C Among the situations you classified as "Likely," which one has a higher level of likelihood? Justify your answer.

View Expected Response

The first bag has a higher level of likelihood, because there is a "similar" number of red and blue candies, while in the third bag there are "many more" candies of other colors than red ones.

Suggested Teacher Guidance – Activity 3

Objective
Assess the likelihood of an observation in a random situation when all relevant information is known and no subjective judgments are involved.

Group Work

Present the birthday party situation and organize students into groups to work on the activity questions. Emphasize that, unlike the previous activity, in this case all relevant information is available, so their assessments should be based on that data and not on personal impressions (e.g., "I'm always lucky, so it's certain I'll get the red candy").

Whole-Class Sharing

Facilitate a whole-class discussion so students can share their responses.
Make sure they:

- Reach consensus on the level of likelihood they assign to each bag (What level of likelihood did you assign to each bag in terms of drawing a red candy: impossible, likely, or certain? Is there any bag where it's impossible to get a red candy?).
- Justify the level of likelihood assigned, based on the number of red candies and/or candies of other colors, that is, based on the available information about the situation (What information did you use to decide which bag had a higher or lower likelihood of drawing a red candy?).
- Distinguish between situations in which chance is involved and those that are impossible or certain. (In which bags would you say chance is involved? Does chance play a role in bags where getting a red candy is impossible or certain? Why?)

Conclusions

Summarize the key points from the discussions and connect students' responses to the following ideas:

- There are situations in which it is possible to estimate the level of likelihood more accurately when all relevant information is available. In these cases, the assessment of the likelihood level does not depend on personal opinions. For example, in question 1, the colors of all the candies in each bag are known, so the likelihood of each observation is assessed objectively

Anticipated Responses and Suggestions

In question b), some students may not understand what the expression "How does chance play a role?" refers to. In those cases, you can clarify that the purpose of the question is for them to identify in which situations chance is involved and why. In particular, they are expected to recognize that chance is present when the outcome is neither certain nor impossible, i.e. when there is uncertainty about what will happen.

Exit Ticket

At the same birthday party, there is also a piñata filled with 20 red candies and 25 blue ones.

Imagine that, with a bit of luck, you manage to collect 26 candies after the piñata bursts.

Which of the following alternatives describes an observation in which chance is present?

a. The candies you collected include both colors
b. All the candies you collected are the same colors
c. There are more red candies than blue ones among the candies you collected

Check possible understandings

Assessment Indicator	Identify whether a given situation is random or not.

Choice	Correct answer	Possible understandings behind the error
a		They do not recognize that the situation is certain (since drawing 26 candies will always include at least one of each color), and therefore they associate it with a "likely" observation in which chance is involved.
b		They do not recognize that the situation is impossible (since drawing 26 candies will always include at least one of each color), and therefore they associate it with a "likely" observation in which chance is involved.
c	X	

Lesson Summary

Lesson 2 - Probability Unit

In this lesson:

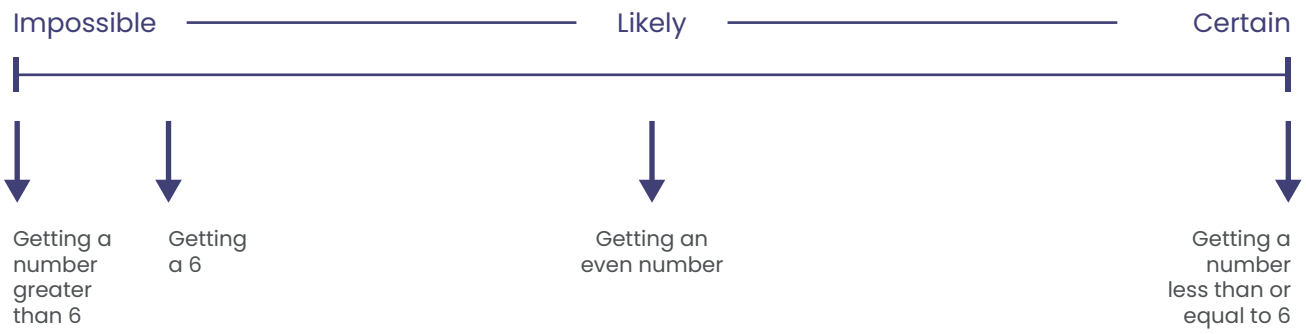
- We learned to qualitatively assess the **likelihood level** of an observation from a random situation using the terms “impossible,” “likely,” and “certain.”

Situation: Measuring the noise level in the schoolyard

- The noise level is null during recess: practically impossible
- The noise level is louder during recess than during class: highly likely
- The noise level is very loud during recess: almost certain

- We used a diagram to **visually represent the level of likelihood**, where observations that are more likely to occur are placed closer to “Certain,” and those that are less likely are placed closer to “Impossible.”

Rolling a die and observing the number on its top face



We recognized that there is **implicit subjectivity** when assessing the level of likelihood, which depends on the perceptions and experiences of the person making the observation.

In the situation "Getting an A on the next math test," two people may assess the level of likelihood differently based on their self-perception in the subject and the grades they have received in the past.

- We introduced the idea that by **considering more information** about a situation, it is possible to reduce the subjectivity involved in assessing the likelihood of an observation.

In the situation "Determining how many days are left until the next eclipse," a person who does not understand the physical and mathematical laws that govern the movement of celestial bodies might assess it as "impossible," while another person who does have that knowledge would say it is "certain."

Mathematical Terms I Can Now Use

- Likelihood levels
- Certain, Likely, Impossible